Comparison and Evaluation of Boundary Conditions for the Absorption of Guided Waves in an FDTD Simulation

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***Abstract*—Several absorbing boundary conditions suitable for terminating the finite-difference time-domain (FDTD) simulation of microstrip structures are compared via numerical experiments. Both a super-absorbing boundary and a modified form of Litva's dispersive boundary condition are found to produce significantly lower reflection than the traditional first-order Mur boundary. The sensitivity of these boundary conditions to the choice of the input parameters, particularly the effective dielectric constant ε_{eff}, is investigated, and optimal choices of these parameters are given.**

I. Introduction

The FDTD METHOD is being increasingly used to characterize integrated circuit packages for microwave and digital circuit applications. As these are typically open region problems, absorbing boundary conditions (ABC's) must be employed to terminate the computational space. Experience shows that even small reflections from these boundaries can introduce significant errors in the computation, and, consequently, a highly absorbing boundary condition is essential to obtaining accurate results.

Two different types of absorbing boundaries are needed to model open, guided-wave structures such as microstrips. One of these should be designed to absorb the waves along the side walls, which are primarily evanescent, while the other must absorb guided waves impinging upon the end walls. Since the fields are exponentially attenuating away from the microstrip conductor in the transverse plane, one can use a Mur type of boundary condition on the side walls provided that these boundaries are sufficiently distant; hence these boundaries pose no special problems. However, as the microstrip line is a dispersive structure, the walls terminating the computational domain in the longitudinal direction must be capable of absorbing normally incident waves with a reasonably wide range of propagation velocities. This communication will concentrate on the boundary conditions used to terminate these microstrip end walls.

II. Description and Comparison of Boundary Conditions

The most commonly used ABC's in FDTD analysis are the first- and second-order Mur conditions [1]. Since the tangential derivatives required by the second-order Mur condition become undefined as one crosses the conductor and dielectric boundaries forming the microstrip, it is almost two orders of magnitude more reflective than the first-order Mur condition when used on microstrip end walls. The first-order Mur condition is a good absorber for normally incident waves traveling at a single speed; however, as the microstrip line is a dispersive structure, a signal containing a range of frequencies has associated waves travelling with a range of speeds. The super-absorbing boundary algorithm [2], [3] applies a boundary condition to both the E-field and the H-field nodes at a boundary as a first step. If the normal velocity of the approaching waves can be estimated, an error-cancellation procedure can then be used to greatly reduce the reflection. A dispersive boundary condition (DBC) can be constructed such that it perfectly absorbs incident waves at two different velocities and which, consequently, has a lower reflection coefficient than the first-order Mur over a wider frequency band [4], [5]. The DBC described in [4] is barely stable at dc, and, therefore, it does not absorb low-frequency signal components well. This problem is alleviated by the addition of a small constant α_1 to one of the two factors. The modified boundary condition is

$$\left(\frac{\partial}{\partial z}+\frac{1}{\nu_1}\frac{\partial}{\partial t}+\alpha_1\right)\left(\frac{\partial}{\partial z}+\frac{1}{\nu_2}\frac{\partial}{\partial t}\right)E=0. \tag{1}$$

In discretized form on the $z = M\Delta z$ boundary, this becomes

$$\begin{aligned} E_M^n =&(\beta+1)E_{M-1}^{n-1}-\beta E_{M-2}^{n-2} \\ &-\gamma_1\gamma_2\left(E_M^{n-2}-2E_{M-1}^{n-1}+E_{M-2}^{n}\right) \\ &-(\beta\gamma_2+\gamma_1)\left(E_{M-1}^{n-2}-E_{M-2}^{n-1}\right) \\ &+(\gamma_1+\gamma_2)\left(E_M^{n-1}-E_{M-1}^{n}\right), \end{aligned} \tag{2}$$

where superscripts denote the time step, subscripts denote the z index and

$$\rho_i=\frac{\nu_i\Delta t}{\Delta z} \qquad \gamma_i=\frac{1-\rho_i}{1+\rho_i(1+\alpha_i\Delta z)}$$
$$\beta=\frac{1+\rho_1}{1+\rho_1(1+\alpha_1\Delta z)}. \tag{3}$$

Note that α_2 is implicitly zero in the formula for γ_i.

Fig. 1 compares the reflections generated by four different boundary terminations: 1) first-order Mur; 2) first-order Mur with super-absorption applied; 3) Litva's original DBC; and,

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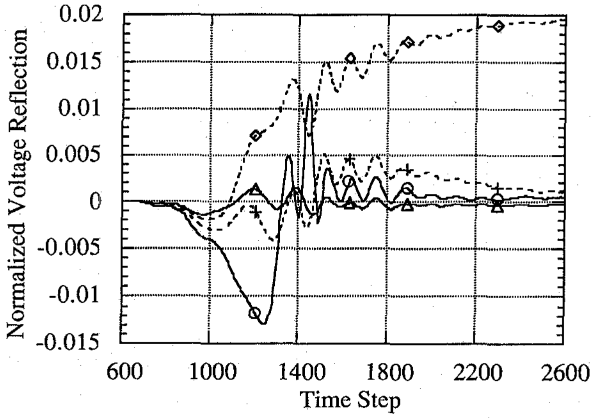

Fig. 1. Comparison of voltage reflections from a microstrip terminated by various ABC's. The reflection is normalized to the peak of the incident wave. Legend: —⊖— first-order Mur with $\varepsilon_{r,\text{eff}} = 8.3$; —△— super-absorbing first-order Mur with $\varepsilon_{r,\text{eff}} = 9.6$; - -◇- - Litva's DBC with $\varepsilon_{r,\text{eff1}} = 7.12$ and $\varepsilon_{r,\text{eff2}} = 8.5$; – + – the modified DBC of this paper with $\varepsilon_{r,\text{eff1}} = 7.12$, $\varepsilon_{r,\text{eff2}} = 8.5$ and $\alpha_1 = 0.1/\Delta z$.

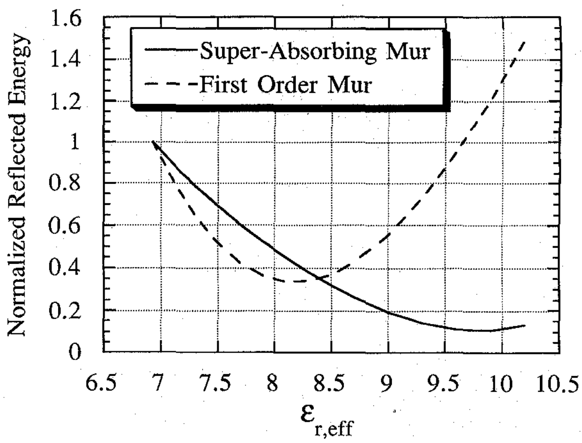

Fig. 2. Variation of normalized reflected energy from super-absorbing and first-order Mur boundaries on a microstrip end as a function of $\varepsilon_{r,\text{eff}}$. Microstrip parameters: $\varepsilon_r = 10.2$; $W = 2.3$ mm; $h = 2.54$ mm. Energy is defined as voltage squared. Note that when these curves are denormalized the super-absorbing curve is 29.9 times lower than the first-order Mur curve.

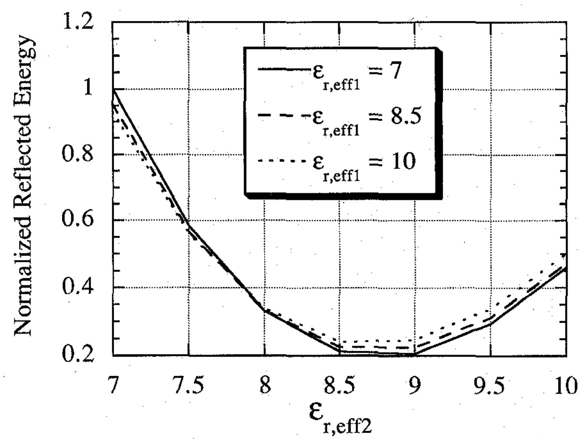

Fig. 3. Influence of $\varepsilon_{r,\text{eff1}}$ and $\varepsilon_{r,\text{eff2}}$ on reflection from a modified DBC boundary with $\alpha_1 = 0.1/\Delta z$. Microstrip parameters: $\varepsilon_r = 10.2$; $W = 2.3$ mm; $h = 2.54$ mm. When denormalized these curves are 4.6 times lower than the first-order Mur curve of Fig. 2.

4) the modified DBC previously given. To permit a fair comparison, the adjustable parameters of each boundary condition, such as ε_{eff}, are set to their optimal values. The geometry analyzed is a microstrip with a relative dielectric constant of 10.2, a dielectric thickness of 2.54 mm and a strip width of 2.3 mm. The incident waveform is a wide-band Gaussian pulse with a 3-dB frequency of 8 GHz. Clearly the super-absorbing algorithm applied to the first-order Mur condition provides the best performance. The modified DBC is also highly absorbing, while the poor absorption of low-frequency components by the original DBC leads to a significant dc offset in the reflected waveform.

III. Choice of Adjustable Parameters

All of the boundary conditions above contain at least one adjustable parameter. The first-order Mur and the first-order Mur with super-absorption both require a choice for the incident velocity of the waves, or equivalently ε_{eff}. Fig. 2 shows how the relative energy in the reflected wave varies as a function of $\varepsilon_{r,\text{eff}}$. The super-absorbing algorithm makes the first-order Mur boundary condition much less sensitive to the value of ε_{eff} and performs best with very high values of ε_{eff}. The quasi-static value of $\varepsilon_{r,\text{eff}}$ as given by Gupta [6] is 6.93. When using the first-order Mur condition one should choose $\varepsilon_{r,\text{eff}}$ to be approximately 20% greater than the quasi-static value, while the optimal value for the super-absorbing first-order Mur is only slightly less than the ε_r of the substrate.

In the modified DBC three parameters must be chosen: two values of ε_{eff} and the value of the constant α_1. The boundary condition is relatively insensitive to the value of α_1. A value of about $0.1/\Delta z$, where Δz is the step size in the direction normal to the boundary, is optimal, but any value between $0.05/\Delta z$ and $0.3/\Delta z$ will yield results almost as good. Negative values of α_1 make the boundary condition unstable, which is understandable when one realizes that this boundary condition corrects the dc problems by forcing the E field to behave as $\exp(j\omega - \alpha_1)t$. Adding constants α_i to both factors rather than to only one also degrades performance.

Fig. 3 shows how the energy reflected by the modified DBC varies as a function of $\varepsilon_{r,\text{eff1}}$ and $\varepsilon_{r,\text{eff2}}$ when $\alpha_1 = 0.1/\Delta z$. Clearly the boundary condition is more sensitive to changes in $\varepsilon_{\text{eff2}}$ than in $\varepsilon_{\text{eff1}}$. Furthermore, for optimal results, $\varepsilon_{r,\text{eff1}}$ should be near the quasi-static value, while $\varepsilon_{r,\text{eff2}}$ should be approximately halfway between the quasi-static value and the substrate ε_r.

When applied to the DBC, the super-absorbing boundary algorithm actually reduces the boundary's effectiveness.

IV. Conclusion

This letter studies the effectiveness of various absorbing boundary conditions in terminating the guiding ends of a microstrip FDTD simulation. The super-absorbing algorithm applied to a first-order Mur boundary has the best performance and the modified DBC given in this letter also performs well. The original DBC is less absorbing than the traditional first-order Mur condition. The sensitivity of these boundary conditions to various parameters is examined and guidelines for choosing these parameters are provided.

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